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ABSTRACT

To assess the productivity of 167 Houston elementary schools, researchers applied a fractional linear programming procedure called Data Envelopment Analysis (DEA) to each school's inputs and outputs. School output was measured by third- and sixth-graders' scores on the Iowa Test of Basic Skills (ITBS). Input resources comprised pupil characteristics, including their previous year's ITBS scores, minority status, lunch payments, and attendance; and school characteristics, such as expenditures per pupil, teacher experience and attendance, and special programs. The use of DEA removed most major problems associated with earlier measures of educational productivity. By solving the fractional linear program associated with each school, the researchers obtained efficiency ratings for individual schools as well as information showing which resources were being underutilized. Eighty-nine schools proved to be efficient relative to all the schools examined. The measurements correlated perfectly with district administrators' identification of outstanding schools. These results provided managers with information to use in guiding system planning, reporting to the public, measuring schools' managerial efficiency, targeting output goals, and identifying areas in which better input utilization might improve productivity. (Author/RW)

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A FRACTIONAL PROGRAMMING MODEL FOR DETERMINING THE EFFICIENCY OF DECISION MAKING UNITS

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ABSTRACT

This paper presents an application of fractional linear programming to assess productivity of 167 elementary schools in the Houston Independent School District. By solving a fractional program associated with each school, efficiency ratings were obtained along with information indicating which resources were being under utilized. This management information, previously unavailable, provided the following as a basis for guiding system planning and reporting to the general public: (1) a measure of the managerial efficiency of individual school units, (2) targeted output goals and identification of priorities in output adjustment, and (3) identification of promising areas for increasing productivity through better input utilization.

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I. INTRODUCTION

In the decade of the 1970's a pervasive push for accountability in public education in the United States [18] brought about revolutionary changes in the management problems faced by administrators of schools. These forces came to bear with swift and urgent impact on large metropolitan school districts such as the Houston Independent School District, a complex organization with 16,000 employees, 194,000 pupils, and an annual budget of $320 million. Rapid population growth, federal regulatory agency mandates and court orders implementing civil rights legislation [11], combined with declining birth rates and alleged white flight to suburban schools, created a set of problems for which few manangement science techniques have been developed and tested. The one that the Board of Education felt was most important, and that provided the occasion for the research reported here, was the need for management information concerning the relative productivity of the 241 individual schools in relation to the resources allocated and the environmental factors affecting their outputs.

For five years, 1975-1979, the administration published an annual report of achievement scores for each school along with input data such as class size, professional staff characteristics, per pupil expenditures, ethnic group enrollment, and socio-economic status [10]. However, no attempt was made to relate these inputs to the outputs of each school. Then in May 1979, in an effort to increase public confidence in Houston public schools,

the Board of Education published the Comparison of Academic Performance Data for Students in the Houston Independent School District and Other Public School Systems [24]. This was a bold and unprecedented step, but the report lacked a comprehensive model for making the comparison, relying instead upon a simple comparison of mean achievement in schools, controlling for socio-economic level.

The need for a better quantitative model became imperative later that year when the state legislature adopted a law requiring statewide competency testing of pupils in all Texas schools and stipulated that "... student performance data shall be aggregated by campus and district and made available to the public, with appropriate interpretations..." [23].

1.1 Problem Description

Out of the context described above, the analysts on this study defined the problem to be:

> *Computation of a single index of managerial efficiency for each school that would provide a measure of the school's ability to produce valued outputs from their inputs.*

In addition to the efficiency measure, it was desired to compute the extent to which inputs were non-productive in inefficient school units, and to determine the input and output values if, in a comparative sense, efficient production were to be attained.

The solution to this problem would provide management with the following information:

1) The managerial efficiency of individual schools -- the productivity of the professional staff of the school given the conditions under which they operate.
2) Targeted output goals and identification of needed input modifications.
3) Identification of areas in which productivity could be increased through improved teaching processes and incentives to faculty members.

1.2 Deficiencies in Prior Methods Employed

We have been unable to find any methods, other than the one used in this investigation, that provide an operational definition of the efficiency of a school. There is, however, a considerable body of literature documenting attempts to define production functions in education. Levin [20] and Hanushek [12] characterize current methodologies as being deficient both conceptually and in their implementation. Following one such attempt, Levin [19] concluded, "The analysis ... is fraught with difficulties that are unusually severe given the present analytical state of the art."

The method most commonly employed is least squares linear regression with a single output as the criterion and a number of predictor variables as input. A variant is the use of multiple

regression to estimate parameters for a linear programming model. A major limitation of all such regression-based methods is that a single production function is sought for all schools through a determination of average production relationships.

Some major criticisms of the methods that have previously been used to develop production functions may be summarized as:

(i) Multiple outputs are not taken into account simultaneously (Boardman, Davis and Sanday [4]).

(ii) Regression coefficients do not necessarily indicate the most efficient way to produce an impact on outputs (Bowles [6]).

(iii) There is a lack of agreement on the meaning of economic terms such as "technical efficiency" when they are applied to schools (Levin [20]).

(iv) The interdependency of inputs (multi-collinearity) and the choice of inputs may produce misleading results (Bowles [6], Bowles and Levin [5]).

(v) The output-input relationships may not be linear or independent (Levin [20], Bowles [6].)

(vi) None of the production function studies are based upon controlled experiments in which inputs have been manipulated (Averch et. al. [1]).

In contrast to the above criticisms, the method employed in this investigation produces a production function for each school while simultaneously accounting for multiple outputs. The need to specify a priori weights is avoided since weights are based upon empirically-derived extremal relations, rather than on average observations. Also, independence of inputs is not assumed. The weak causality inference is, however, an unresolved problem for all non-experimental research, including the methods employed in the present study.

1.3 Notation and Conventions

In this exposition, matrices and sets are denoted by upper case Latin letters. Lower case Latin letters that are underlined denote column vectors. The zero vector is denoted by $\underline{0}$, a vector of all ones by $\underline{1}$, and the identity matrix by I. A' and b' denote the transposition of matrix A and column vector $\underline{b}$, respectively. Scalars are denoted by lower case Latin and Greek letters.

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II. MATHEMATICAL MODEL

In this section we present the mathematical programming model used in our analysis of the Houston Independent School District. This type of model was first proposed by Charnes, Cooper, and Rhodes [8], and can be used for a variety of situations in which a manager wishes to obtain some measure of the efficiency of a large number of relatively independent units (such as schools). This type of analysis is called Data Envelopment Analysis, and henceforth will be referred to as DEA.

Let s denote the total number of schools that are under consideration for analysis. For each school, suppose we have data on o outputs (such as reading achievement scores) and i inputs (such as per pupil expenditures in dollars). Let $\underline{o}_j$ denote the o-component vector of outputs and let $\underline{i}_j$ denote the i-component vector of inputs for school j. For this analysis we assume that $\underline{o}_j > \underline{0}$ and $\underline{i}_j > \underline{0}$ for all j.

Using the above notation, school j can be characterized by the vector, $[\underline{o}_j', \underline{i}_j']$. One measure that could be used to compare schools is the sum of the outputs divided by the sum of the inputs; that is, $(\underline{o}_1' \underline{1})/(\underline{i}_1' \underline{1}), \ldots, (\underline{o}_s' \underline{1})/(\underline{i}_s' \underline{1})$. However, since the units of measure for both the inputs and outputs may not be compatible, weights are required to transform the inputs and outputs to unitless measures. Let $\underline{u}$ denote the o-component vector of nonnegative weights for the outputs and $\underline{v}$ denote the i-component vector of nonegative weights for the inputs. We then define a weighted ratio for school k, $r(k, \underline{u}, \underline{v}) = (\underline{o}_k' \underline{u})/(\underline{i}_k' \underline{v})$.

The question is how to select the weights $\underline{u}$, $\underline{v}$. Charnes, Cooper, and Rhodes [8], suggest that the set of weights for each school be selected in a manner that is most favorable for that school. Let W_k be a compact nonempty set of weights for school k defined as follows: $W_k = \{(\underline{u}, \underline{v}) : (\underline{o}_j' \underline{u})/(\underline{i}_j' \underline{v}) \leq 1, j = 1, \ldots, s, \underline{o}_k' \underline{u} = 1, \underline{u} \geq \underline{0}, \underline{v} \geq 0\}$. Therefore, $r(j, \underline{u}, \underline{v})$ is finite over W_j for $j = 1, \ldots, s$. We say that a set of weights $(\underline{u}_k, \underline{v}_k) \in W_k$ are most favorable for school k if $r(k, \underline{u}_k, \underline{v}_k) \geq r(k, \underline{u}, \underline{v})$ for all $(\underline{u}, \underline{v}) \in W_k$. If $r(k, \underline{u}_k, \underline{v}_k) = 1$, then school k is called *efficient*; that is, no other school produced more of the outputs per unit of inputs employed. Otherwise, school k is called *inefficient*. This condition is closely related to what has been called a *Pareto optimum* in the economics literature (see [8]).

To obtain a set of weights most favorable for school k, we solve the following nonlinear program:

$$\max \quad h_k = r(k, \underline{u}, \underline{v})$$

$$\text{s.t.} \quad (\underline{u}, \underline{v}) \in W_k.$$

Due to the definition of W_k, it is more convenient to solve the related problem,

$$\min \quad z_k = \frac{1}{h_k} = \frac{1}{r(k, \underline{u}, \underline{v})} \tag{1}$$

$$\text{s.t.} \quad (\underline{u}, \underline{v}) \in W_k \,, \tag{2}$$

which may be written as the linear program

$$\min \quad z_k = \underline{i}_k' \underline{v} \tag{3}$$

$$\text{s.t.} \quad \underline{i}_j' \underline{v} - \underline{o}_j' \underline{u} \geq 0, \quad j = 1, \ldots, s \tag{4}$$

$$\underline{o}_k' \underline{u} = 1 \tag{5}$$

$$\underline{v}, \underline{u} \geq \underline{0}. \tag{6}$$

Note that from (4) and (5) we have $\underline{i}_k' \underline{v} \geq \underline{o}_k' \underline{u} = 1$. Since s (the number of constraints) is usually much larger than i + o (the number of variables) it is usually easier to solve the dual of (3) - (6) which may be written as follows:

$$\left.\begin{array}{l} \max \quad z_k \\ \text{s.t.} \\ \quad \sum_{j=1}^{s} \begin{bmatrix} \underline{i}_j \\ \hline -\underline{o}_j \end{bmatrix} \lambda_j + \begin{bmatrix} \underline{0} \\ \hline \underline{o}_k \end{bmatrix} z_k + \begin{bmatrix} \underline{x}_1 \\ \hline \underline{x}_2 \end{bmatrix} = \begin{bmatrix} \underline{i}_k \\ \hline \underline{0} \end{bmatrix} \\ \qquad \underline{\lambda} \geq 0, \ \underline{x}_1 \geq \underline{0}, \ \underline{x}_2 \geq \underline{0} \end{array}\right\} P_k$$

where slack variables $\underline{x}_1$ and $\underline{x}_2$ have been appended. We call $h_k = 1/z_k$ the efficiency index for school k. Hence, if school k is efficient, then $z_k = h_k = \lambda_k = 1$, and all other variables equal 0. Otherwise, $z_k > 1$, $h_k < 1$ and the optimal slacks, $\underline{x}_1^*$, may be interpreted as the resources (inputs) that are not being utilized efficiently.

In addition, z_k and optimal slacks $\underline{x}_1^*$ and $\underline{x}_2^*$, may be used to calculate an adjusted input vector, $\hat{\underline{i}}_k$, and an adjusted output vector, $\hat{\underline{o}}_k$, that would make school k efficient. The adjusted vectors (as determined in [8]) are given by

$$\hat{\underline{i}}_k = \underline{i}_k - \underline{x}_1^*, \text{ and}$$

$$\hat{\underline{o}}_k = z_k \underline{o}_k + \underline{x}_2^*.$$

A computer program was written to solve $P_1, \ldots, P_s$ in a single run via the revised simplex method. For each school the program produces the optimal z_k, h_k, $\underline{x}_1$, $\underline{x}_2$, $\underline{u}$, $\underline{v}$, $\hat{\underline{i}}_k$, $\hat{\underline{o}}_k$, and the results of ranging on the right-hand-side. The program begins with the initial feasible solution $\lambda_k = z_k = 1$ and all other variables zero. The basis inverse is maintained in product form and only the non-zeroes are stored. The reinversion routine is based on the work of Hellerman and Rarick [15]. This routine incorporates a technique known as "splitting the bump" (see Kalan [16] and Orchard-Hays [21]) and uses the "spike swapping theory" of Helgason and Kennington [13]. A good description of the computational procedures may be found in Helgason [14]. The arrays and working files require approximately $(3s + 13)(o + i) + 4s$ words of core storage. The program is described and documented in [17].

III. SELECTION OF OUTPUTS AND INPUTS

The Iowa Test of Basic Skills (ITBS), a comprehensive standarized test of achievement, had been administered to all elementary school pupils in Houston in May, 1978. Among other measures, the ITBS provided a composite score for the aggregated sub-test scores of each pupil; the mean of the composite score for grade three and the mean for grade six were used as the two output measures for each school.

This high level of aggregation -- over pupils and over sub-tests -- seemed adequate for the purpose of the study. If detailed individual school planning information were required, sub-test scores would be the relevant measures, with the scores aggregated by classroom. It should be noted also that available data restricted the analysis to cognitive outputs, although there are other school outputs that are valued.

The twelve input measures were selected from those that had been reported by the Houston administration in their annual Elementary School Profiles [10]. These included:

(A) *Measures of the characteristics of pupils that were highly correlated with achievement scores:*

(1 and 2) The previous year's achievement scores on the same test battery for the preceding grade levels -- 2nd and 5th grades,

(3) percent non-minority enrollment[1],

(4) percent of students paying full lunch price, and

(5) percent attendance: average daily attendance ÷ average daily membership

(B) *Measures of school resources available:*

(6) number of professionals per 100 pupils,

(7) local and state expenditures per pupil,

(8) federal money allocated per pupil,

(9) number of special programs in the school,

(10) percent of teachers with masters degrees,

(11) percent of teachers with more than 3 years experience, and

(12) the percent of teaching days that the assigned teachers were present in the classroom; an average over all teachers assigned to the school; part-time teachers were prorated.

These were available measures regularly collected by the school administration. They were aware that there were other desirable input measures, but chose to go ahead with the analysis to gain experience with the method which, if feasible and useful, would be repeated with more carefully designed measures.

IV. RESULTS

4.1 Output Tables and Interpretation

For the inputs and outputs described above, we calculated the inefficiency rating, Z_k, for 167 elementary schools in the Houston Independent School District.[2] This meant solving 167 linear programs each having 14 rows and 182 columns. Of these schools, 46.7% (78) were found to be inefficient. In order to display both efficiency and effectiveness, a graph relating efficiency and third grade achievement (see Figure 1) and a graph relating efficiency and sixth achievement (see Figure 2) were prepared. In order to present complete results and preserve the readability of the graphs, the following method was employed.

1) A school's number was used to indicate its associated point on the graph, thus providing identification of the school unit along with the measurement.

2) Inefficient schools that had the same efficiency rating and output measurement as some other school already on the graph were listed at the bottom of the figure, along with the number of the comparable school. For example, schools 25 and 73 both have $h_k = .91$ and third grade achievement of 3.5. School 25 is shown on the graph in Figure 1 and 73 is paired with 25 at the bottom of Figure 1.

Figures 1 and 2 About Here

Schools are commonly classified as above or below some achievement norm. However, it can be seen from Figures 1 and 2 that low achieving schools may be efficient. Consider efficient school 87. Given its very low outputs (2.9 and 5.2), it might seem that this school was not doing a good job. However, given its inputs, school 87 did as well as any school in the district; much better than inefficient school 49. Even though these two schools had very similar outputs, one was efficient and the other inefficient. From Table 1, it can be seen that school 49 did not fully utilize many of its inputs, the most notable being $604/pupil of slack local and state funds. Thus appropriate planning and administrative response with respect to these two schools would be quite different.

Note that relative efficiency is displayed in Figures 1 and 2 (horizontal axis) along with normed measures of output (vertical axis). Thus, both effectiveness and efficiency may be considered together. No measures of absolute efficiency are provided, of course, but school administrators and board members are accustomed to measuring how well their schools perform on achievement tests relative to a norm. With the additional information provided by DEA, they are able to assess the relationship of these effectiveness indicators to the quantity of resources provided. Thus, a school that performs poorly with respect to achievement scores would be undesirable, even if efficient, but the management response would be different than it would be if the school were inefficient. This is discussed more fully in the following section.

Table 1 About Here

The information provided by DEA can be configured in many different ways. Since space is limited, it was decided simply to illustrate by including two possibilities.[3] Table 1 focuses on the most inefficient schools in the district. All these schools appear to have had adequate resources if fully utilized. Inputs such as minority group enrollment and socio-economic status are not shown in the tables since these are not alterable by management decision or intervention. (They may affect the efficiency, however, and in actual practice would be displayed. They are deleted here to make the tables more readable.)

Table 2 highlights high-achieving, near-efficient schools. It might be that some reallocatable inputs from these schools could be transferred to efficient low-achieving schools with no deterimental effect on the outputs of these schools. Considerations such as the above are employed in management applications discussed in the following section.

Table 2 About Here

4.2 Administrative Implications

The individual school is the production unit in a large school district. The General Superintendent and his central staff can increase productivity only indirectly -- through hiring and assignment of personnel and provision of resources and incentives that have the potential for increasing production if they are efficiently employed. As

a result, information that the central administration may use to guide system planning is a crucial, and often missing, factor in top-level management of schools.

DEA provides this management information. The General Superintendent and his staff require school principals to submit annual operational plans in which they set school goals and specify needed resources. These operational plans may employ DEA results as follows: (i) as information for school principals to use in their proposed plans, (ii) as management "audit" information for use in reviewing these plans before approval, (iii) as system scanning information to balance scarce resources among schools, and (iv) for annual review and evaluation of accomplishment of the previous year's plan.

Let us examine briefly the DEA information used for each of these applications.

(i) In developing an operating plan for the individual school, the principal of an efficient school can increase output goals if achievement is below the norm and will be able to request additional resources for inputs that have opportunity costs. The principal of an inefficient school is able to identify those inputs that are unproductive. Furthermore, he can specify within-school plans for utilizing inputs more effectively in order to increase outputs to the level of expected achievement if the unit were efficient.[4]

(ii) For management audit of proposed plans, the General Superintendent is able to use efficiency, inefficiency, slack, and opportunity cost information for such purposes as: determining whether low achievement schools are setting realistic improvement goals, whether addi-

tional input requests are justified, and what inputs are not being fully utilized. Without DEA results, such review cannot be done except in the most general way based upon average allocations and achievement norms.

(iii) As a means of balancing resources, the central staff can identify where slack resources exist in inefficient schools with high output. These slack resources might be better utilized in efficient schools with low output.

(iv) At the completion of each year, use of DEA results can show whether goal achievement is as expected and whether input increments previously authorized were efficiently used. An overall audit of the system is possible to determine whether the proportion of efficient schools is increasing or decreasing.

Since the analysis compares each school to the observations obtained on all other schools, it follows that an element of competitiveness is involved. If an efficient school, through better operational plans succeeds in raising its achievement more than others, then some schools which were formerly efficient might become inefficient. Likewise, some inefficient schools might be reduced to even greater inefficiency. Competition is seen to be desirable by school administrators with whom we have worked so long as it does not unfairly pernalize those schools which have many obstacles to overcome. Fairness of results is evidenced by the number of efficient schools in Figures 1 and 2 which have low achievement because of high proportions of children from disadvantaged families. We also found that faculties of schools who had been

striving for better results were pleased to have an objective measure of their program.

V. SUMMARY AND CONCLUSION

The General Superintendent of Houston's school system found the pilot testing of DEA in 1979 to be sufficiently interesting so that he had the results distributed to area superintendents and to school principals. In addition, the District authorized a second analysis using 1980 results of the new mandatory statewide competency test of basic skills. This analysis will be used to gain additional experience with the methodology.

The validity of DEA was assessed in an informal test in which the General Superintendent and his staff first identified "trouble" schools and "outstanding" schools. In the two hour session approximately 40 of the 167 schools were reviewed in detail and a check of administrative assessment against DEA solutions resulted in 100% correct classification; i.e. the "trouble" schools were all inefficient to some degree and the outstanding schools were efficient. Equally important, the reasons for a school's status based on known local conditions generally coincided with the DEA slack values.

The three major problems we encountered in this pilot study were (1) obtaining data to specify adequate input measures, (2) obtaining data to specify outputs that were not limited to cognitive test results, and (3) difficulties in communicating the results of a complex quantitative process to those affected by the results. It is hoped that these problems will become less severe with increasing experience.

Other applications of DEA in schools are foreseen in Houston and in other school districts where research is in progress. Using the individual pupil as the unit of analysis, DEA can identify pupils making

inefficient use of their resources for learning; this information can be aggregated at the classroom level to measure management efficiency of teachers in producing a learning environment.

This methodology is appropriate for use in non-school applications as well. For example, it is now being applied to highway safety units. Traffic violations and convictions are being used as outputs and the number and assignment of patrol units as inputs. Other potential uses for not-for-profit enterprises include the efficiency analysis of welfare units, emergency medical service units, and post office mail delivery.

Table 1 Schools Less Than 90% Efficient

School		Outputs		Alterable Slack Resources (Inputs)							
h_k	k	3rd grade	6th grade	Profess per 100 pupils	Local + State $/pupil	Fed $ per pupil	Stu. Attend-ance (%)	Tea. Attend-ance (%)	Tea. with Masters Degrees (%)	Tea. with > 3 yrs. exp (%)	# Special Programs
.82	23	3.1	5.2	2.7	$370	$ 44	2.8%	6.1%	5.0%	8.6%	5.0
.83	12	3.3	5.7	0	$ 24	$ 10	0	5.3%	4.5%	17.7%	1.3
.83	30	3.1	5.3	1.4	$ 28	$ 93	4.0%	0	2.2%	15.8%	0
.84	49	3.0	5.1	4.0	$604	$107	1.9%	4.8%	0	0	.5
.85	72	3.1	5.5	.1	0	$ 36	3.4%	2.2%	9.6%	0	1.3
.86	139	3.3	5.4	.5	$ 60	$ 82	0	1.5%	8.9%	18.8%	1.5
.86	155	3.2	5.8	0	$ 34	0	0	3.2%	0	5.8%	0
.88	116	3.4	5.2	.1	$ 74	0	6.4%	3.9%	23.4%	17.1%	3.1
.88	148	3.8	6.6	0	0	0	1.4%	0	2.1%	8.4%	.3
.89	138	3.5	6.4	0	$145	$ 16	0	.8%	0	0	0
.89	70	3.2	5.6	.8	0	0	4.9%	5.4%	0	0	.4
.89	44	3.1	5.2	2.0	$250	$176	4.0%	7.6%	0	25.6%	3.7
.89	165	3.8	6.5	0	0	0	.4%	0	4.5%	29.2%	0
.89	136	3.7	6.4	.6	$ 70	$ 35	1.5%	3.4%	11.5%	0	0

Table 2 High-Achieving Near-Efficient Schools

School		Outputs		Reallocatable Slack Resources (Inputs)			
h_k	k	3rd grade	6th grade	Profess/ 100 pupils	Local + State $/pupil	Teachers with Masters Degrees (%)	Teachers with 3 years experience (%)
.94	11	4.3	7.2	.4	0	13.3%	28.8%
.95	158	4.3	7.5	0	$ 50	0	23.2%
.96	100	4.4	7.2	1.3	$326	18.0%	0
.96	133	4.5	7.3	.1	0	0	0
.96	119	4.5	7.5	0	$ 99	20.1%	18.2%
.97	145	3.6	7.1	0	0	6.0%	0
.97	63	4.7	7.5	.7	$326	32.7%	0
.98	52	4.3	7.3	0	$ 93	5.3%	0
.98	154	4.5	7.5	0	$ 33	11.5%	14.9%
.99	129	4.7	8.0	0	$ 93	7.9%	18.6%

Appendix A Sample Planning Form

School 139 $h_{139} = .860$

Measurement Name	Observed Measurement	Measurement if Efficient	Measurement Goal
Output			
3rd Grade (1978)	3.3 Grade Equiv	3.8	
6th Grade (1978)	5.4 Grade Equiv	6.3	
Input			
2nd Grade (1977)	2.8 Grade Equiv	2.7	
5th Grade (1977)	4.6 Grade Equiv	4.5	
% Non-minority	1.7 %	1.7	
% Pay lunch	22.0 %	22.0	
# Prof./100	5.9 prof/100 pupil	5.4	
Federal $/pupil	$210/pupil	$128	
Local-State $/pupil	$1185/pupil	$1125	
% Attendance	90.8 %	90.8	
% Teacher days worked	94.5 %	93.0	
% Masters degree	44.0 %	35.1	
% More 3 yrs. exp.	69.0 %	50.2	
# special programs	6 special programs	4.5	

Program Objectives

FOOTNOTES

1. Note that inputs are defined in directionality so that increasing units of input would be expected to yield increasing units of output.
2. Only elementary schools that had been in operation more than two years were included.
3. For other variants, see Bessent and Bessent [3].
4. See Appendix A for a sample planning form for use by principals. Planning documents of this type have proved to be helpful with respect to communication. Using the form displayed, the principal would observe that 2nd grade achievement in 1977 was at the norm (2.8), yet these same children a year later in 3rd grade were .5 below norm (3.3). Further, there seems to be a cumulative loss in 4th, 5th, and 6th grades as 5th is 1.2 below norm and 6th is 1.4. Thus, a concentrated effort to improve achievement in the intermediate grades is indicated; that is, the 6th grade needs to "catch up". School faculty should examine their special programs, purchases with instructional dollars and the utilization of instructional personnel as the slack values indicate that these resources are not being effectively used.

REFERENCES

1. Averch, H. A., S. J. Carrol, T. S. Donaldson, H. J. Kiesling, and J. Pincus, How Effective is Schooling?, Educational Technology Publications, Englewood Cliffs, N. J., (1974).

2. Banker, R., Charnes, A., Cooper, W. W., and Schinnar, A., "A Bi-Extremal Principle for Estimation of Efficiency Frontier Parameter Values," Research Report CCS 374 (revision of CCS 329 of March 1979), Center for Cybernetic Studies, The University of Texas, (April 1980).

3. Bessent, A. and W. Bessent, "Determining the Comparative Efficiency of Schools Through Data Envelopment Analysis," Educational Administration Quarterly, 16, 2, (1980), 57-75.

4. Boardman, A. E., O. A. Davis and P. R. Sanday, "A Simultaneous Equations Model of the Educational Process," Journal of Public Economics, 7, 1, (1977), 23-49.

5. Bowles, S. and H. M. Levin, "The Determinants of Scholastic Achievement -- A Critical Appraisal of Some Recent Evidence," Journal of Human Resources, 3, 4, (1968), 3-24.

6. Bowles, S., "Towards an Education Production Function," in Education, Income, and Human Capital, Editor, Hansen, W. Lee, National Bureau of Economic Research, New York, N. Y., (1970), 61-70.

7. Charnes, A. and Cooper, W. W., "Management Science Relations for Evaluation and Management Accountability," Research Report CCS 341, Center for Cybernetic Studies, The University of Texas at Austin, (May 1979). Journal of Enterprise Management, 4, (1980). (to appear in Management Science (1980)).

8. Charnes, A., W. Cooper, and E. Rhodes, "Measuring the Efficiency of Decision Making Units," European Journal of Operational Research, 2, 6, (1978), 429-444.

9. Charnes, A., Cooper, W. W., and Schinnar, A., "Transforms and Approximations in Cost and Production Function Relations," Research Report CCS 339, Center for Cybernetic Studies, The University of Texas at Austin, (March 1979).

10. Elementary School Profiles 1977-78, Houston Independent School District, Houston, Texas, (1978), page 1.

11. Halperin, S., "The Educational Arena," Educational Evaluation and Policy Analysis, 2, 1, (1980), 27-36.

12. Hanushek, E. A., "Conceptual and Empirical Issues in Estimation of Educational Production Functions," Journal of Human Resources, 14, 3, (1979), 351-387.

13. Helgason, R. V., and J. L. Kennington, "Spike Swapping in Basis Reinversion," (to appear in Naval Research Logistics Quarterly).

14. Helgason, R. V., "A Lagrangean Relaxation Approach to the Generalized Fixed Charge Multicommodity Minimal Cost Network Flow Problem," unpublished dissertation, Southern Methodist University, Dallas, Texas, (1980).

15. Hellerman, E. and D. Rarick, "Reinversion With the Preassigned Pivot Procedure," Mathematical Programming, 1, (1971), 195-216.

16. Kalan, J. E., "Aspects of Large-Scale In-Core Linear Programming," Proceedings of the ACM, (1971), 304-313.

17. Kennington, J., "A Primal Simplex Code for Computing the Efficiency of Decision Making Units," Technical Report OREM 80001, Department of Operations Research, Southern Methodist University, Dallas, Texas, (1980).

18. Lessinger, L., Every Kid A Winner: Accountability in Education, Simon and Schuster, New York, N. Y., (1970).

19. Levin, M., "Measuring Efficiency in Educational Production," Public Finance Quarterly, 2, (1974), 3-24.

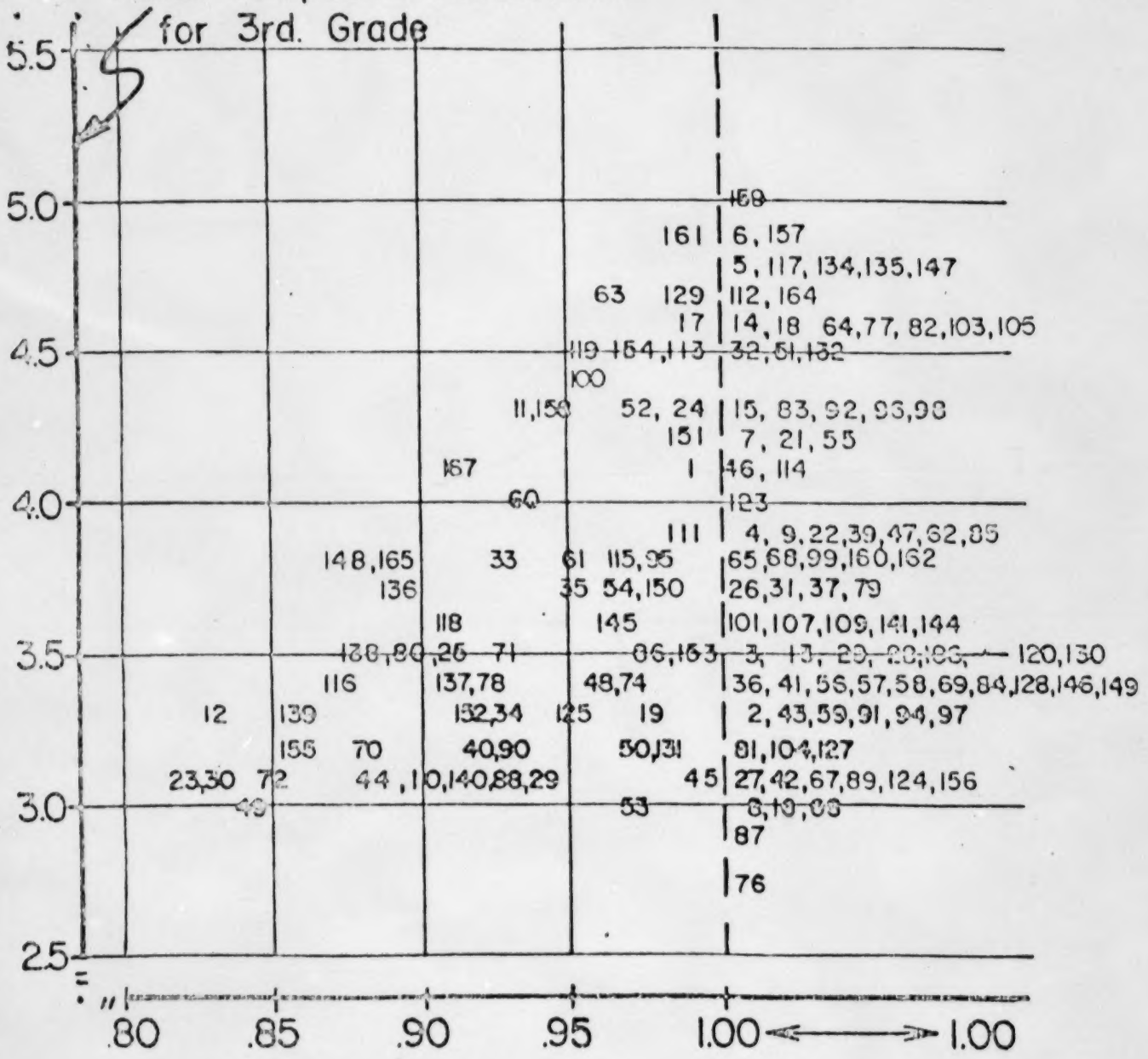
20. Levin, M., "Concepts of Economic Efficiency and Educational Production," in Education as an Industry, Editors J. Froomkin, D. T. Jamison, and R. Radner, Ballinger Publishing Co., Cambridge, (1976).

21. Orchard-Hays, W., "An Exercise in Basis Inversion," SIGMAP Newsletter, No. 15, (1973).

22. Rhodes, E. L., "Data Envelopment Analysis and Related Approaches for Measuring the Efficiency of Decision-Making Units with an Application to Program Follow Through in U.S. Education," unpublished doctoral dissertation (Pittsburgh, PA: School of Urban and Public Affairs, Carnegie-Mellon University, 1978).

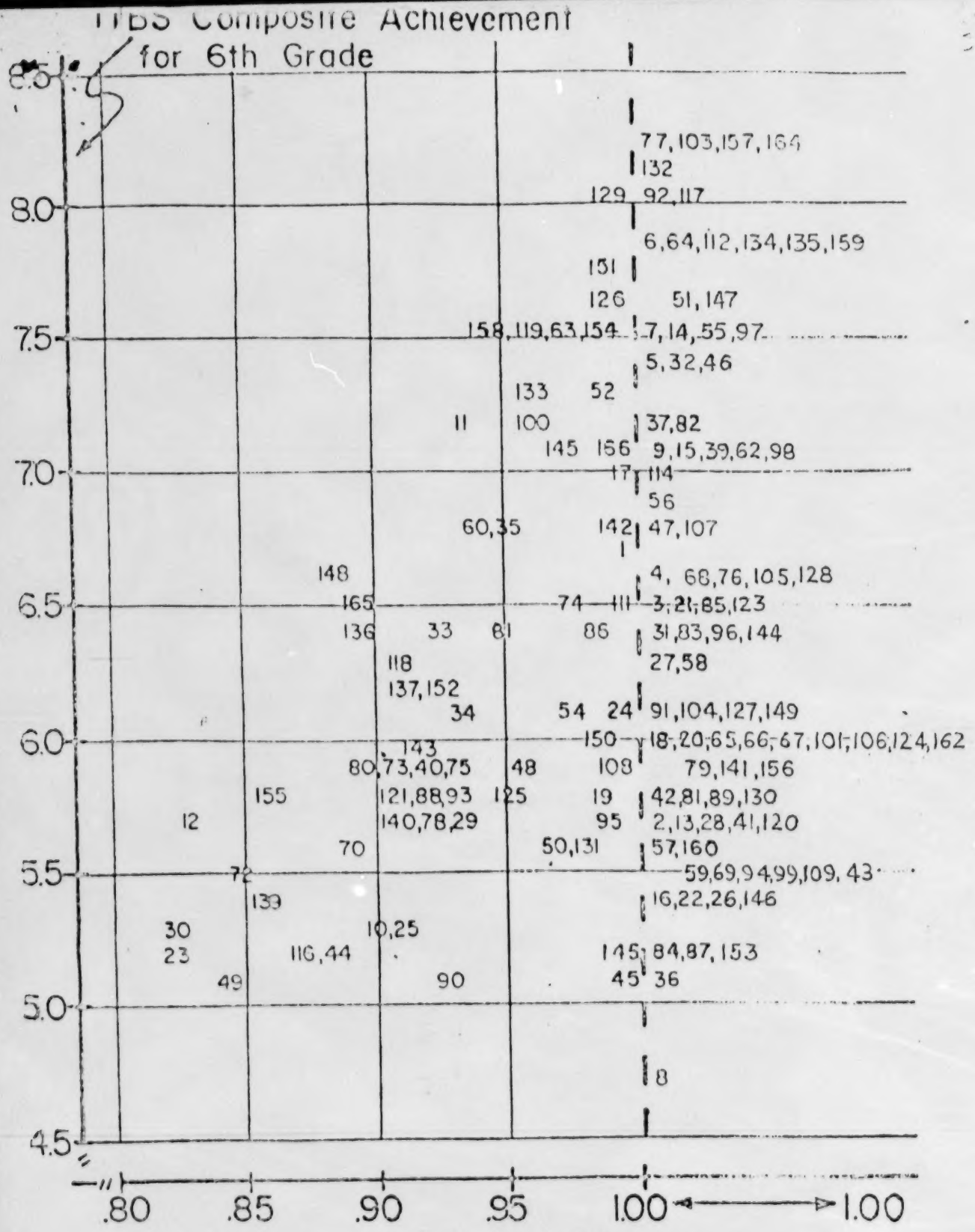
23. Texas Education Code, Section 16.176, Amended by 1979 Texas Legislature, Senate Bill 350.

24. Thompson, B., L. Miller, E. Say, and J. Forsbach, Comparison of Academic Performance Data for Students in the Houston Independent School District and Other Public School Systems, Houston Independent School District, Houston, Texas, (1979).



Inefficient School not on Plot	38	73	75	93	102	106	110	121	122	126	133	142	143	163	166	
Same as School Numbered	129	25	33	90	88	153	61	80	11	129	119	1	20	44	113	

Fig. 1 Efficiency and Third Grade Achievement



Inefficient School not on Plot	38	53	71	102	110	113	115	122	138	161	163	167	
Same as School Numbered	154	52	33	40	35	17	52	60	136	38	44	118	

Fig. 2 Efficiency and Sixth Grade Achievement